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A Predictive Control Based Approach to Networked Control Systems with Input Nonlinearity: Design and Stability Analysis

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***Abstract*— In this paper, a predictive control based approach is proposed for a networked control system containing a nonlinear input process. The approach uses a two-step predictive controller to deal with the input nonlinearity and a delay and dropout compensation scheme to compensate for the communication constraints in a networked control environment. Theoretical results are presented for the closed-loop stability of the system. Simulation examples illustrating the validity of the approach are also presented.**

***Keywords*— Networked control systems, Predictive control, Nonlinear input process, Two-step approach, Delay and dropout compensation scheme**

I. Introduction

The research on "Networked Control Systems" (NCSs) has been an emerging trend in recent years [1]–[3]. The limits to the performance of control systems in a networked control environment are caused primarily by networked-induced delays and data packet dropout [4]. These communication constraints can mean in NCSs that the control signal for the plant is delayed even unavailable, which results in an open loop. The desire to obtain a better performance than that resulting from holding the last available control signal or using zero control during open loop intervals in NCSs, has led to a model based control architecture [5] and to a predictive control based control architecture [6]–[9]. The key idea of the model based approach is that the knowledge of the plant dynamics is used to reduce the usage of the network, while in the predictive control based approach proposed in [7], the plant dynamics is further used to produce future control signals to actively compensate for the random network-induced delay in the forward channel actively with the use of a corresponding time delay compensator at the actuator side. A better performance can be expected since the predictive control based approach takes greater advantage of the knowledge available.

In this paper, following the predictive control based approach in [7], we extend its application to a NCS with a nonlinear input process and random network-induced delays in both forward and backward channels and data packet dropout in the forward channel. Using the two-step design approach [10], we first apply the predictive control method to the linear part of the system considered in this paper to generate the intermediate control predictions using delayed sensing data and previous control information. This process is distinct from [7] in that only the available previous information is used to make the predictions. The real control predictions for the system are then obtained from the nonlinear input relationship assuming the inverse of the static nonlinear function can be calculated numerically. In order to compensate for the network-induced delays in both channels and data packet dropout as well, a Delay and Dropout Compensation Scheme (DDCS) is designed, which consists of two components, a matrix selector at the controller side to compensate for the network-induced delay in the backward channel and a delay compensator at the actuator side to compensate for the network-induced delay and data packet dropout in the forward channel (see Fig.1 for the whole structure). The implementation of DDCS makes the predictive control based approach work well in a network-based environment. The stability theorem of the closed loop system is obtained by placing a sector constraint on the nonlinearity due to the inaccuracy of calculating the real control predictions. Simulations are also done to illustrate the validity of the approach.

The remainder of this paper is organized as follows. The design of the proposed approach is presented in Section 2. Then the theoretical results for the system stability and the simulation results are presented in Section 3 and Section 4, respectively. The paper gives the conclusions in Section 5.

II. Design of Networked Predictive Control System with Input Nonlinearity

The following Single-Input-Single-Output (SISO) system S with a nonlinear input process is considered in this paper,

$$S : \begin{cases} x(k+1) = Ax(k) + bv(k) & \text{(1a)} \\ y(k) = cx(k) & \text{(1b)} \\ v(k) = f(u(k)) & \text{(1c)} \end{cases}$$

where $x \in \mathbb{R}^n$, $u, v, y \in \mathbb{R}$, and $f(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$ is a memoryless static nonlinear function.

In this section, we present first the design details of the two-step predictive control approach to system S and then the design of DDCS to compensate for the network-induced delays and data packet dropout when such a system is implemented in a networked control environment.

A. Design of the two-step predictive control approach

The key idea of the two-step predictive control approach is to design an intermediate control signal $v(k)$ of the linear part of system S (equations (1a) and (1b)) with a linear predictive control method (A Linear Generalized Predictive Control (LGPC) method is adopted in this paper) first, and then obtain the real control signal $u(k)$ for system S from the nonlinear relationship $v(k) = f(u(k))$ (see [9], [10] for its applications to a Hammerstein model). The design details of the two steps in a networked control environment are presented respectively as follows.

1) Design of LGPC: In LGPC, a quadratic objective function is normally adopted, represented by:

$$J(N_1, N_2, N_u) = \sum_{j=N_1}^{N_2} q_j(\hat{y}(k+j|k-\tau_{sc,k}) - \omega(k+j))^2 + \sum_{j=1}^{N_u} r_j \Delta v^2(k+j-1) \quad (2)$$

where N_1 and N_2 are the minimum and maximum prediction horizons, N_u is the control horizon, $q_j, N_1 \leq j \leq N_2$ and $r_j, 1 \leq j \leq N_u$ are weighting factors, $\omega(k+j), j = N_1, ..., N_2$ are the set points, $\Delta v(k) = v(k) - v(k-1)$ is the control increment and $\hat{y}(k+j|k-\tau_{sc,k}), j = N_1, ..., N_2$ are the forward predictions of the system outputs, which are obtained on data up to time $k - \tau_{sc,k}$ and will be calculated in detail later, where $\tau_{sc,k}$ is the network-induced delay in the backward channel at time k.

Let $\bar{x}(k) = [x^T(k) \ v(k-1)]^T$, then system S can be represented by S',

$$S' : \begin{cases} \bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{b}\Delta v(k) & (3a) \\ y(k) = \bar{c}\bar{x}(k) & (3b) \end{cases}$$

where $\bar{A} = \begin{pmatrix} A & b \\ 0 & 1 \end{pmatrix}$, $\bar{b} = \begin{pmatrix} b \\ 1 \end{pmatrix}$, $\bar{c} = \begin{pmatrix} c & 0 \end{pmatrix}$. Thus the j' step forward output prediction at time k' is

$$\hat{y}(k'+j'|k') = \bar{c}\bar{A}^{j'}\bar{x}(k') + \sum_{l'=0}^{j'-1} \bar{c}\bar{A}^{j'-l'-1}\bar{b}\Delta v(k'+l')$$

Let $j' = j + \tau_{sc,k}$, $k' = k - \tau_{sc,k}$, $l' = l + \tau_{sc,k}$, then the forward output predictions at time k based on the information of the state on time $k - \tau_{sc,k}$ and control signals from time $k - \tau_{sc,k} - 1$ is

$$\hat{y}(k+j|k-\tau_{sc,k}) = \bar{c}\bar{A}^{j+\tau_{sc,k}}\bar{x}(k-\tau_{sc,k}) + \sum_{l=-\tau_{sc,k}}^{j-1} \bar{c}\bar{A}^{j-l-1}\bar{b}\Delta v(k+l) \quad (4)$$

In [7], the previous control signals $v(k-1), ..., v(k-\tau_{sc,k})$ are used to calculate the predictive control sequence at time k. However, this information is actually not available for the controller in practice due to the random network-induced delay in the forward channel. As will be discussed further in Section II. B, in a networked predictive control environment, a sequence of future control signals is packed to send to the actuator, and the actuator only picks out one from the sequence according to the specific time delay in the forward channel. Therefore the controller does not know the real control signal adopted by the actuator unless it receives the information about the previous control signals applied to the actuator. Only in such a special case, with no delay in the forward channel, the previous control signals are all known by the controller immediately. Therefore, in this paper, we develop a new method to deal with this problem, in which only the control and output information before time $k - \tau_{sc,k}$ is used to generate the predictive control sequence, by including the control sequence from time $k - \tau_{sc,k}$ to $k - 1$ as part of the predictive control sequence.

Let $\hat{Y}(k|k-\tau_{sc,k}) = [\hat{y}(k+N_1|k-\tau_{sc,k}) \ \cdots \ \hat{y}(k+N_2|k-\tau_{sc,k})]^T$, $\Delta V'(k|k-\tau_{sc,k}) = [\Delta v(k-\tau_{sc,k}|k-\tau_{sc,k}) \ \cdots \ \Delta v(k+N_u-1|k-\tau_{sc,k})]^T$, then

$$\hat{Y}(k|k-\tau_{sc,k}) = E_{\tau_{sc,k}}\bar{x}(k-\tau_{sc,k}) + F_{\tau_{sc,k}}\Delta V'(k|k-\tau_{sc,k}) \quad (5)$$

where $F_{\tau_{sc,k}}$ is a $(N_2 - N_1 + 1) \times (N_u + \tau_{sc,k})$ matrix with the non-null entries defined by $(F_{\tau_{sc,k}})_{ij} = \bar{c}\bar{A}^{N_1+\tau_{sc,k}+i-j-1}\bar{b}, j - i \leq N_1 + \tau_{sc,k} - 1$, and $E_{\tau_{sc,k}} = [(\bar{c}\bar{A}^{N_1+\tau_{sc,k}})^T \ (\bar{c}\bar{A}^{N_1+\tau_{sc,k}+1})^T \ \ldots \ (\bar{c}\bar{A}^{N_2+\tau_{sc,k}})^T]^T$. Note here that $E_{\tau_{sc,k}}$ and $F_{\tau_{sc,k}}$ vary with different $\tau_{sc,k}$s.

Let $\varpi_k = [\omega(k+N_1) \ \cdots \ \omega(k+N_2)]^T$, then the optimal predictive control increments from k to $k + N_u - 1$ can be calculated by letting $\partial J(\cdot)/\partial \Delta V' = 0$,

$$\Delta V(k|k-\tau_{sc,k}) = M_{\tau_{sc,k}}(\varpi_k - E_{\tau_{sc,k}}\bar{x}(k-\tau_{sc,k})) \quad (6)$$

where $\Delta V(k|k-\tau_{sc,k}) = [\Delta v(k|k-\tau_{sc,k}) \ \cdots \ \Delta v(k+N_u-1|k-\tau_{sc,k})]^T$, $M_{\tau_{sc,k}} = H_{\tau_{sc,k}}(F^T_{\tau_{sc,k}}QF_{\tau_{sc,k}} + R)^{-1}F^T_{\tau_{sc,k}}Q$, Q, R are diagonal matrices with $Q_{i,i} = q_i, R_{i,i} = r_i$ respectively and $H_{\tau_{sc,k}} = [0_{N_u \times \tau_{sc,k}} \ I_{N_u \times N_u}]$, $I_{N_u \times N_u}$ is the identity matrix with rank N_u.

Remark 1 Normally, the minimum prediction horizon can be set as 1. Rewrite the maximum prediction horizon N_2 as N_p. The following constraint between N_u and N_p needs to be always held in order to implement the LGPC method successfully,

$$N_u \leq N_p \quad (7)$$

2) The nonlinear input process: If the nonlinear function $f(\cdot)$ is invertible then its inverses $f^{-1}(\cdot)$ exists such that

$$\Delta u(k) = f^{-1}(\Delta v(k)) \quad (8)$$

Thus, at every time instant k, the intermediate control increments $\Delta v(k), k = 1, 2, \cdots, N_u$ can be obtained from (6), and then the real control increments $\Delta u(k), k = 1, 2, \cdots, N_u$ can be calculated from (8) thus enabling the control law to be derived for system S'.

If $\Delta u(k)$ can be calculated accurately using (8), thus enabling the function $f^{-1}(\cdot)$ to be exactly known, then the system with compensation for the nonlinear input process is equivalent to LGPC and the system is stable if and only if the linear part of system S with LGPC is stable. However, in practice, it is usually impossible to calculate $u(k)$ that accurately. This inaccuracy introduces to the LGPC a nonlinear disturbance, which makes the stability analysis difficult.

Denote the real inverse of $f(\cdot)$ by $\hat{f}^{-1}(\cdot)$ and for simplicity of notation, let $\hat{\vec{f}}^{-1}(\cdot) : \mathbb{R}^{N_u} \to \mathbb{R}^{N_u}$ with $\hat{\vec{f}}^{-1}(\Delta V(k|k-\tau_{sc,k})) = [\hat{f}^{-1}(\Delta v(k|k-\tau_{sc,k})) \ \cdots \ \hat{f}^{-1}(\Delta v(k+N_u-1|k-\tau_{sc,k}))]^T$. Then from the discussion above, the real predictive control increment sequence for system S can be represented by

$$\Delta U(k|k-\tau_{sc,k}) = \hat{\vec{f}}^{-1}(\Delta V(k|k-\tau_{sc,k}) \quad (9)$$

where $\Delta U(k|k-\tau_{sc,k}) = [\Delta u(k|k-\tau_{sc,k}) \ \cdots \ \Delta u(k+N_u-1|k-\tau_{sc,k})]^T$.

B. Design of DDCS

To enable the two-step predictive control approach to work appropriately in a networked control environment, a Delay and Dropout Compensation Scheme (DDCS) is proposed in this section.

The following assumptions are first made for the DDCS design:

A1. Each data packet containing the sensing data is sent with a time stamp to notify when it was sent from sensor to controller. This enables the network-induced delay in the backward channel for each data packet known to the controller. This information is then used to calculate the appropriate control predictions;

A2. At every time instant k, the control predictions $\Delta U(k|k-\tau_{sc,k})$ are packed into one data packet with time stamps k and $\tau_{sc,k}$. These time stamps are to notify the time when it was sent and also the network-induced delay in the backward channel which the control predictions were based on. The control sequence is then sent to the actuator simultaneously thus enabling the network-induced delays in both channels for each control predictive sequence known to the actuator;

A3. The sum of the maximum network-induced delay in the forward channel (noted by $\bar{\tau}_{ca}$) and the maximum number of continuous data packet dropout (noted by $\bar{\chi}$) is bounded by the control horizon, i.e.,

$$\bar{\tau}_{ca} + \bar{\chi} \leq N_u - 1 \quad (10)$$

Based on the assumptions above, the two components of the DDCS, the matrix selector and the delay compensator, which are to deal with the network-induced delay in the backward channel and the network-induced delay and data packet dropout in the forward channel respectively, are presented in the following sections.

1) Compensation for the random network-induced delay in the backward channel - a matrix selector: Note the fact that the calculation complexity of the predictive control increments (equation (6)) seriously depends on the network-induced delay in the backward channel τ_{sc} since the matrices $E_{\tau_{sc,k}}, F_{\tau_{sc,k}}, M_{\tau_{sc,k}}, H_{\tau_{sc,k}}$ vary with this delay at time k, i.e., $\tau_{sc,k}$. Thus for the online implementation, it is a great burden for the controller to calculate the predictive control increments if τ_{sc} varies over a large range. However, these matrices, actually, can be calculated off line since all the matrices are fixed for a given τ_{sc}. This advantage enables us to calculate off line all the matrices with respect to the specific τ_{sc}s, store them in a device called the "matrix selector" and just choose the appropriate ones from the matrix selector when calculating online the predictive control increments, according to the current value of the delay $\tau_{sc,k}$, which is known to the controller recalling assumption A1.

Let $\mathscr{E}_{sc} = \{E_0, E_1, \cdots, E_{\bar{\tau}_{sc}}\}$, $\mathscr{F}_{sc} = \{F_0, F_1, \cdots, F_{\bar{\tau}_{sc}}\}$, $\mathscr{M}_{sc} = \{M_0, M_1, \cdots, M_{\bar{\tau}_{sc}}\}$, $\mathscr{H}_{sc} = \{H_0, H_1, \cdots, H_{\bar{\tau}_{sc}}\}$, where $\bar{\tau}_{sc}$ is the upper bound of the network-induced delay in the backward channel, then we have for any k (or $\tau_{sc,k}$), $E_{\tau_{sc,k}} \in \mathscr{E}_{sc}$, $F_{\tau_{sc,k}} \in \mathscr{F}_{sc}$, $M_{\tau_{sc,k}} \in \mathscr{M}_{sc}$, $H_{\tau_{sc,k}} \in \mathscr{H}_{sc}$, respectively. For a practical implementation, these matrices are calculated off line and stored in the matrix selector for online use.

2) Compensation for the random network-induced delay and data packet dropout in the forward channel - a delay compensator: As presented in assumption A2, the predictive control increment sequence $\Delta U(k|k-\tau_{sc,k})$ is sent to the actuator all in one data packet. When a new sequence arrives at the actuator side, it is compared with the one already in the so called "delay compensator" according to the time stamps (which notify the time when the sequences were sent from the controller) and only the one with the latest time stamp is stored. The delay compensator is specially designed for the actuator and it can only store one control sequence (data packet) at any time. For example, denote the sequence arrives at the actuator side as $\Delta U(k_1|k_1-\tau_{sc,k_1})$ with a time stamp k_1 and the one already in the delay compensator is $\Delta U(k_2|k_2-\tau_{sc,k_2})$ with a time stamp k_2. Then if $k_1 > k_2$, $\Delta U(k_2|k_2-\tau_{sc,k_2})$ will be replaced by $\Delta U(k_1|k_1-\tau_{sc,k_1})$; otherwise $\Delta U(k_1|k_1-\tau_{sc,k_1})$ will be simply discarded and the delay compensator remains unchanged.

The comparison process is introduced at the actuator side due to the fact that different data packets may experience different delays in the forward channel, thereby producing a situation where for example a data packet sent earlier from the controller may arrive at the actuator later or may never arrive in the case of data packet dropout. As a result of the comparison process, the predictive control sequence stored in the delay compensator is always the latest one available at any specific time.

As for the actuator, it can be either time-driven or event-driven. At every execution time instant[1], the actuator picks out

[1]A fixed time interval between two successive time instants for time-driven actuator, while variable for event-driven actuator since the execution time instant is trigged by the event that a new predictive control sequence is stored in the delay compensator and thus can be random.

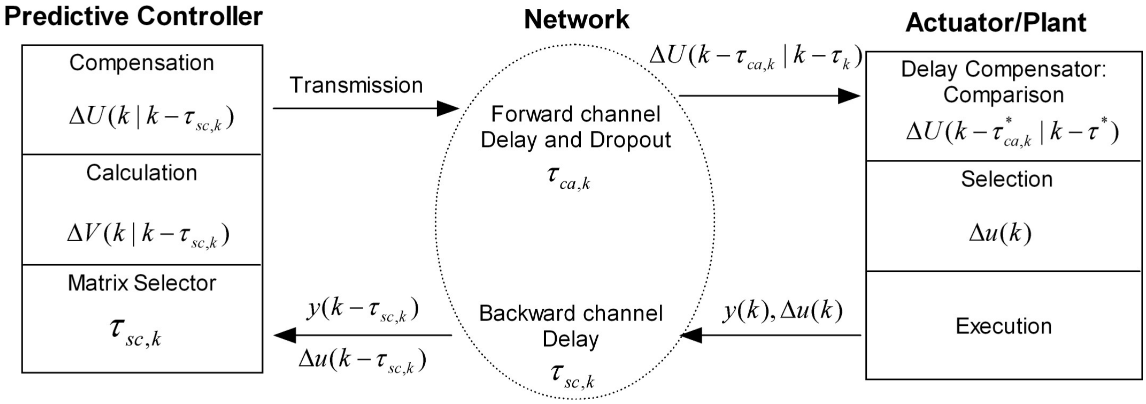

Fig. 1. The structure of networked predictive control system with input nonlinearity

the appropriate control increment signal which can compensate for current network-induced delay in the forward channel from the predictive control increment sequence in the delay compensator and applies it to the plant. The method to choose the appropriate control increment signal at a specific time will be explained in detail in the next section. It is necessary to point this out that the appropriate control increment is always available using the delay compensator if assumption A3 holds.

The two-step predictive control approach with DDCS can now be summarized by the following four steps:

S1. **Calculation.** The predictive controller calculates the intermediate predictive control increment sequence $\Delta V(k|k-\tau_{sc,k})$ through (6) with the use of the proposed matrix selector and delayed information of states and control signals. The predictive control increment sequence $\Delta U(k|k-\tau_{sc,k})$ is then obtained by compensating for the nonlinear input process using (9);

S2. **Transmission.** $\Delta U(k|k-\tau_{sc,k})$ is packed and sent to the actuator simultaneously with time stamps k and $\tau_{sc,k}$;

S3. **Comparison.** The delay compensator updates its information according to the time stamps once a data packet arrives;

S4. **Execution.** An appropriate control increment signal is picked out from the control sequence in the delay compensator and applied to the plant.

The structure of the predictive based approach with DDCS (so called "Networked Predictive Control Systems"(NPCSs)) is illustrated in Fig.1.

III. Stability Analysis

In this section, the closed loop formulation of such a NPCS with a nonlinear input process is derived, and then the stability theorem is obtained using switched system theory under a sector constraint of the nonlinearity due to calculation inaccuracy.

A. Closed loop system

Let $\tau^*_{ca,k}$ denote the network-induced delay in the forward channel of the predictive control increment sequence, from which the control signal is picked out by the actuator at time instant k. The time when the sequence was sent from the controller side can then be read from its time stamp as

$$k^* = k - \tau^*_{ca,k} = \max_j\{j|\Delta U(j|j-\tau_{sc,j}) \in \Gamma_k\} \tag{11}$$

where Γ_k is the set of the predictive control increment sequences that are available during time interval $(k-1,k]$ at the actuator side, including not only the one in the delay compensator but others that arrive at the actuator during this interval (see Fig.2). From equations (9), (11), the control signal adopted by the actuator at time k is obtained as

$$\Delta u(k) = d^T_{\tau^*_{ca,k}} \Delta U(k-\tau^*_{ca,k}|k-\tau^*_k) \tag{12}$$

where $d_{\tau^*_{ca,k}}$ is a $N_u \times 1$ matrix with all entries 0 except the $(\tau^*_{ca,k}+1)$th is 1, τ^*_k is the RTT (Round Trip Time) with respect to $\tau^*_{ca,k}$, i.e., $\tau^*_k = \tau^*_{ca,k} + \tau^*_{sc,k}$, and $\tau^*_{sc,k} = \tau_{sc,k^*}$.

From equations (6), (9) and noticing for any vector V with an appropriate dimension, $d^T_{\tau^*_{ca,k}} \vec{\hat{f}}^{-1}(V) = \hat{f}^{-1}(d^T_{\tau^*_{ca,k}} V)$ recalling the definition of $\vec{\hat{f}}^{-1}(\cdot)$, thus we obtain (assume the set point $\omega = 0$ without loss of generality)

$$\begin{aligned}\Delta u(k) =& d^T_{\tau^*_{ca,k}} \Delta U(k-\tau^*_{ca,k}|k-\tau^*_k)\\ =& d^T_{\tau^*_{ca,k}} \vec{\hat{f}}^{-1}(\Delta V(k-\tau^*_{ca,k}|k-\tau^*_k))\\ =& \hat{f}^{-1}(d^T_{\tau^*_{ca,k}} \Delta V(k-\tau^*_{ca,k}|k-\tau^*_k))\\ =& \hat{f}^{-1}(-K^*_{\tau,k}\bar{x}(k-\tau^*_k))\end{aligned} \tag{13}$$

where $K^*_{\tau,k} = d^T_{\tau^*_{ca,k}} M_{\tau_{sc,k}} E_{\tau_{sc,k}}$[2]. The real control increment for linear system (1a) and (1b) at time k can then be obtained as

$$\Delta v(k) = f(\Delta u(k)) = f \circ \hat{f}^{-1}(-K^*_{\tau,k}\bar{x}(k-\tau^*_k)) \tag{14}$$

where $f \circ \hat{f}^{-1}(\cdot) = f(\hat{f}^{-1}(\cdot))$ is the composite function of $f(\cdot)$ and $\hat{f}^{-1}(\cdot)$.

Let $X(k) = [\bar{x}^T(k-\bar{\tau}) \ \cdots \ \bar{x}^T(k)]^T$, $w(k) = \Delta v(k)$, then the closed loop system can be represented by

$$S^*: \begin{cases} X(k+1) = \widetilde{A}X(k) + \widetilde{b}w(k) & \text{(15a)}\\ w(k) = f \circ \hat{f}^{-1}(-K^*_{\bar{\tau},k}X(k)) & \text{(15b)}\end{cases}$$

[2]Note that the value of $K^*_{\tau,k}$ varies with the delays in both channels, and thus it has $(\bar{\tau}_{ca}+1)(\bar{\tau}_{sc}+1)$ different values in total.

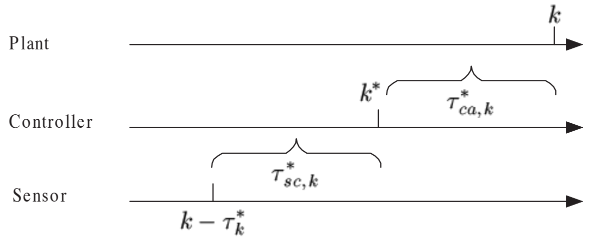

Fig. 2. Time delays of the control signal adopted by the actuator at time k

where $\widetilde{b} = [0_{n+1,1} \ \cdots \ 0_{n+1,1} \ \bar{b}_{n+1,1}^T]^T$, $K_{\bar{\tau},k}^*$ is a $1 \times (\bar{\tau}+1)$ block matrix with block size of $1 \times (n+1)$ and all its blocks 0 except the $(\bar{\tau}+1-\tau_k^*)$th is $K_{\tau,k}^*$ (the set of all the possible $K_{\bar{\tau},k}^*$ will be denoted by $\mathbb{K}$), and $\widetilde{A} =$

$$\begin{pmatrix} 0_{n+1} & I_{n+1} & & & \\ & 0_{n+1} & I_{n+1} & & \mathbf{0} \\ & & \ddots & \ddots & \\ & \mathbf{0} & & 0_{n+1} & I_{n+1} \\ & & & & \bar{A} \end{pmatrix}.$$

B. Stability Analysis

As has been pointed out in Section II.A.2), the compensation for the nonlinear input process using (8) is generally not accurate, and this inaccuracy introduces to the linear part of the system ((1a) and (1b)) a nonlinear disturbance, which appears in the form of $f \circ \hat{f}^{-1}(\cdot)$. Though generally $f \circ \hat{f}^{-1}(\cdot) \not\equiv 1$, it is reasonable to assume that the calculation error meets some accuracy requirement to a certain extent, which results in a sector constraint for the term $f \circ \hat{f}^{-1}(\cdot)$, as described in assumption A4 as follows.

A4. The nonlinearity due to the calculation inaccuracy is supposed to satisfy a sector constraint, i.e., there exist $0 < \underline{\varepsilon} \le \bar{\varepsilon} < \infty$, s.t.

$$\underline{\varepsilon}\alpha \le f \circ \hat{f}^{-1}(\alpha) \le \bar{\varepsilon}\alpha, \forall \alpha \in \mathbb{R} \tag{16}$$

This constraint can be denoted by

$$f \circ \hat{f}^{-1}(\cdot) \in [\underline{\varepsilon}, \bar{\varepsilon}] \tag{17}$$

Notice here that generally $0 < \underline{\varepsilon} \le 1 \le \bar{\varepsilon} < \infty$.

Using assumption A4, we obtain that for any specific $\alpha \in \mathbb{R}$, there exists a real number $\varepsilon_\alpha, \underline{\varepsilon} \le \varepsilon_\alpha \le \bar{\varepsilon}$ such that $f \circ \hat{f}^{-1}(\alpha) = \varepsilon_\alpha \alpha$, equation (15b) can thereby be rewritten as

$$\begin{aligned} w(k) =& f \circ \hat{f}^{-1}(-K_{\bar{\tau},k}^* X(k)) \\ =& -\varepsilon_k K_{\bar{\tau},k}^* X(k) \end{aligned} \tag{18}$$

where $\varepsilon_k \in [\underline{\varepsilon}, \bar{\varepsilon}]$ represents the compensation for the specific nonlinearity for the term $K_{\bar{\tau},k}^* X(k)$ at time k.

Recalling equations (15a) and (18), the closed loop system S^* can then be written as

$$\begin{aligned} X(k+1) =& \widetilde{A}X(k) + \widetilde{b}w(k) \\ =& (\widetilde{A} - \varepsilon_k \widetilde{b} K_{\bar{\tau},k}^*)X(k) \\ =& \Lambda(\varepsilon_k, K_{\bar{\tau},k}^*)X(k) \end{aligned} \tag{19}$$

where the closed loop matrix $\Lambda(\varepsilon_k, K_{\bar{\tau},k}^*) = \widetilde{A} - \varepsilon_k \widetilde{b} K_{\bar{\tau},k}^*$ has the form

$$\Lambda(\varepsilon_k, K_{\bar{\tau},k}^*) = \begin{pmatrix} 0_{n+1} & I_{n+1} & & & \\ & 0_{n+1} & I_{n+1} & & \mathbf{0} \\ & & \ddots & \ddots & \\ & \mathbf{0} & & 0_{n+1} & I_{n+1} \\ \cdots & -\varepsilon_k \bar{b} K_{\tau,k}^* & \cdots & & \bar{A} \end{pmatrix}.$$

The position and value of the term $-\varepsilon_k \bar{b} K_{\tau,k}^*$ depends on the specific delays in the both channels at time k, i.e., $(\Lambda(\varepsilon_k, K_{\bar{\tau},k}^*))_{\bar{\tau}+1,j} = -\varepsilon_k \bar{b} K_{\tau,k}^*$, $j = \tau_k^* = 1, 2, \cdots, \bar{\tau}$, and $(\Lambda(\varepsilon_k, K_{\bar{\tau},k}^*))_{\bar{\tau}+1,\bar{\tau}+1} = \bar{A} - \varepsilon_k \bar{b} K_{\tau,k}^*$, if $\tau_k^* = \bar{\tau}+1$.

Theorem 1 The closed loop system S^* is stable if A4 holds and there exists a positive definite solution $P = P^T > 0$ for the following $2(\bar{\tau}_{ca}+1)(\bar{\tau}_{sc}+1)$ LMIs

$$\Lambda^T(\underline{\varepsilon}, K_{\bar{\tau},k}^*) P \Lambda(\underline{\varepsilon}, K_{\bar{\tau},k}^*) - P \le 0 \tag{20a}$$
$$\Lambda^T(\bar{\varepsilon}, K_{\bar{\tau},k}^*) P \Lambda(\bar{\varepsilon}, K_{\bar{\tau},k}^*) - P \le 0 \tag{20b}$$

where $K_{\bar{\tau},k}^* \in \mathbb{K}$.

Proof. Let $V(k) = X^T(k)PX(k)$ be a Lyapunov function candidate, then the incremental $V(k)$ for system S^* can be obtained using equation (19)

$$\begin{aligned} \Delta V(k) =& X^T(k)(\Lambda(\varepsilon_k, K_{\bar{\tau},k}^*)^T P \Lambda(\varepsilon_k, K_{\bar{\tau},k}^*) - P)X(k) \\ =& X^T(k)(\widetilde{A}^T P \widetilde{A} - P - \varepsilon_k \widetilde{A}^T P \widetilde{b} K_{\bar{\tau},k}^* \\ & - \varepsilon_k K_{\bar{\tau},k}^{*T} \widetilde{b}^T P \widetilde{A} + \varepsilon_k^2 K_{\bar{\tau},k}^{*T} \widetilde{b}^T P \widetilde{b} K_{\bar{\tau},k}^*)X(k) \quad (21) \\ \overset{\text{Def}}{=}& X^T(k) \mathscr{A}(\varepsilon_k, K_{\bar{\tau},k}^*) X(k) \quad (22) \end{aligned}$$

where $\varepsilon_k \in [\underline{\varepsilon}, \bar{\varepsilon}], K_{\bar{\tau},k}^* \in \mathbb{K}$.

Notice that for any $\varepsilon_k \in [\underline{\varepsilon}, \bar{\varepsilon}]$, there exists $0 \le \lambda_k \le 1$ s.t. $\varepsilon_k = \lambda_k \underline{\varepsilon} + (1-\lambda_k)\bar{\varepsilon}$, and thus we obtain by substituting this into (22)

$$\begin{aligned} \mathscr{A}(\varepsilon_k, K_{\bar{\tau},k}^*) =& \lambda_k \mathscr{A}(\underline{\varepsilon}, K_{\bar{\tau},k}^*) + (1-\lambda_k)\mathscr{A}(\bar{\varepsilon}, K_{\bar{\tau},k}^*) \\ & - \lambda_k(1-\lambda_k)(\underline{\varepsilon} - \bar{\varepsilon})^2 K_{\bar{\tau},k}^{*T} \widetilde{b}^T P \widetilde{b} K_{\bar{\tau},k}^* \end{aligned} \tag{23}$$

From equations (20a), (20b) and (22), $\mathscr{A}(\underline{\varepsilon}, K_{\bar{\tau},k}^*)$ and $\mathscr{A}(\bar{\varepsilon}, K_{\bar{\tau},k}^*)$ are semi-negative definite for all $K_{\bar{\tau},k}^* \in \mathbb{K}$. Notice that P is symmetric positive definite, and then $K_{\bar{\tau},k}^{*T} \widetilde{b}^T P \widetilde{b} K_{\bar{\tau},k}^*$ is semi-positive definite as a symmetric matrix, thus enabling $\mathscr{A}(\varepsilon_k, K_{\bar{\tau},k}^*)$ to be semi-negative definite for any $\varepsilon_k \in [\underline{\varepsilon}, \bar{\varepsilon}]$ and $K_{\bar{\tau},k}^* \in \mathbb{K}$, which completes the proof.

Remark 2 It is necessary to point this out that according to assumption A4 and Theorem 1, what is required for the stability of the system is to satisfactorily meet the sector constraint in equation (17), no matter how the inverse function $\hat{f}^{-1}(\cdot)$ is calculated. It implies that the function $f(\cdot)$ does not need to be theoretically invertible as long as its inverse can be obtained by a numerical method and satisfies the sector constraint. One can refer to [11] and the references therein for more information of the calculation of $\hat{f}^{-1}(\cdot)$.

IV. Simulation

In [7], the authors have shown the validity of the predictive based approach for a linear plant with a random delay in the forward channel and a constant delay in the backward channel using simulations and a real test rig as well. In this section, a second order plant model in discrete time as follows is adopted to illustrate the validity of the proposed approach in this paper for the system with a nonlinear input process and random delays in both channels and data packet dropout in the forward channel,

$$A = \begin{pmatrix} 0.98 & 0.1 \\ 0 & 1 \end{pmatrix}, b = \begin{pmatrix} 0.04 \\ 0.1 \end{pmatrix}, c = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

Other parameters of the simulation are chosen as $\bar{\tau} = 8, \bar{\tau}_{ca} = 4, \bar{\tau}_{sc} = 4, N_u = 8, N_p = 10, \underline{\varepsilon} = 0.5, \bar{\varepsilon} = 1.5$, and the initial state $x_0 = [-1 - 1]^T$. The delays in both channels are set to vary randomly within their upper bounds.

The simulation result is illustrated in Fig.3, from which it is seen that the closed loop stability of such a system can be still guaranteed under certain conditions when a nonlinear input process and random delays in both channels are present in the system and compensated for by the proposed approach, though the states vary over a larger range in such a case.

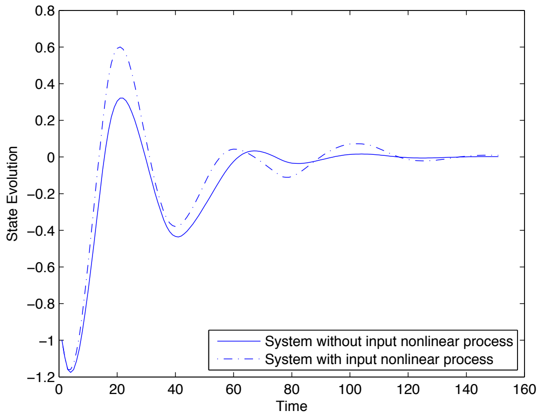

Fig. 3. The validity of the compensation for the input nonlinear process

V. Conclusion

In this paper, a novel approach with the integration of the two-step predictive control method and a delay and dropout compensation scheme is proposed for a networked control system containing a nonlinear input process. In the approach, the predictive controller for the linear part of the system is first designed using delayed sensing data, and the nonlinear input can be viewed as a nonlinear disturbance after a compensation scheme. The communication constraints considered in this paper, i.e., random delays in both channels and data packet dropout in the forward channel, are dealt with by the delay and dropout compensation scheme, which consists of two components configured at both the controller and actuator sides. The stability theorem for the closed loop system is obtained using switched system theory. Simulation work has also been done to illustrate the validity of the approach.

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1. A predictive control-based approach to networked hammerstein systems: Design and stability analysis

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Abstract: In this paper, a predictive control-based approach is proposed for a Hammerstein-type system which is closed through some form of network. The approach uses a two-step predictive controller to deal with the static input nonlinearity of the Hammerstein system and a delay and dropout compensation scheme to compensate for the communication constraints in a networked control environment. Theoretical results are presented for the closed-loop stability of the system. Simulation examples illustrating the validity of the approach are also presented. © 2008 IEEE.
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